Infusing Mathematical Practices into Tasks to Support and Challenge Every Student

Cornell University/NY Master Teacher Program

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@JBayWilliams
If your strip of paper is \( \frac{3}{4} \) of the whole, draw:

1. \( \frac{1}{2} \) of the whole
2. \( \frac{2}{3} \) of the whole
3. \( \frac{3}{2} \) of the whole
4. \( \frac{5}{4} \) of the whole
WELCOME!

Share your...

• name
• school
• role
• what you wish for your students
WHAT DO YOU WISH FOR YOUR STUDENTS?

A. Independent thinkers
B. Problem solvers
C. Excited to take on tough problems
D. Able to determine their own strategy to solve a problem
E. Self-motivated to figure out a problem
F. Other: ____________________
GOALS

- Deepen our understanding of Mathematical Practices.
- Explore a collection of tasks that support mathematical practices.
- Develop strategies for helping students develop the mathematical practices.
I have sort-of lost track of these. I know all 8 by name and what each looks like in practice. I pay attention to these as I daily plan & teach lessons.
Why Mathematical Practices?
Pass out all the cards.

Read your scenario and decide which Mathematical Practice is BEST represented in the scenario.

On your turn, read your vignette to the group, share where you think it goes and place it there.
<table>
<thead>
<tr>
<th>MP 1</th>
<th>MP 2</th>
</tr>
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<tbody>
<tr>
<td>MP 3</td>
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</tr>
<tr>
<td>MP 5</td>
<td>MP 6</td>
</tr>
<tr>
<td>MP 7</td>
<td>MP 8</td>
</tr>
</tbody>
</table>

D. Vignette

A. Vignette

Negotiate!
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tr>
<td>MP 1</td>
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<td>H</td>
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<td>A</td>
<td>MP 4</td>
<td>E</td>
</tr>
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<td>MP 6</td>
<td>F</td>
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<td>MP 7</td>
<td>C</td>
<td>MP 8</td>
<td>G</td>
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**Matching Mini-Vignettes: One Solution Set**
The Relationship of the Mathematical Practices

(Kelemanik, Lucenta, & Creighton, 2016)
Christopher is working on addition strategies. He looks at 8 + 7 and decides to use the context of his toy cars to think about the problem. He recognizes that 8 cars equals 5 cars and 3 more and 7 cars equals 5 cars and 2 more. He pictures the cars lined up in fives and solves the problem by adding 5 + 5 + 5.

Vignette from *Everything you Need for Mathematics Coaching* (McGatha & Bay-Williams, 2018)
Mathematical Practices

The Relationship of the Mathematical Practices
(Kelemanik, Lucenta, & Creighton, 2016)

G. The class is working on integer operations on the number line. They solve problems such as these:

\[-3 + 14 = \quad \quad -15 + 10 = \quad \quad \]
\[21 + (-30) = \quad \quad \]

Kelly notices that whenever the signs are different she moves back to zero and then jumps the rest of the distance (e.g., for \(-3 + 14\), she moves +3 to zero and then up to 11). She notices that this answer looks like the difference of 14 and 3 and uses this idea to solve other problems.

Vignette from Everything you Need for Mathematics Coaching (McGatha & Bay-Williams, 2018)
The Relationship of the Mathematical Practices

(Kelemanik, Lucenta, & Creighton, 2016)
C. In solving for $n$, Amanda notices that in the equation $6(n + 1) - 4(n + 1) = -30$, the expressions inside the parenthesis match, so she can subtract the groups and get $2(n + 1)$ rather than apply the distributive property as a first step.

Vignette from *Everything you Need for Mathematics Coaching* (McGatha & Bay-Williams, 2018)
Mathematical Practices

The Relationship of the Mathematical Practices
(Kelemanik, Lucenta, & Creighton, 2016)
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4. $\frac{5}{4}$ of the whole
A Whole Lot of Fun

1. What Mathematical Practices did you use to solve the tasks?

2. What Mathematical Practices have potential to develop if using this task with students?
TIPS FOR POSING QUESTIONS
TIPS FOR POSING QUESTIONS

• Plural forms
• Tentative language
• Open-ended
• Positive presuppositions
• Higher-order thinking

TIPS FOR POSEING QUESTIONS

• Plural forms

What strategy will you use? vs What strategies will you use?
Tips for Posing Questions

• Tentative language

What will you do first? vs What might be a first step?
TIPS FOR POSING QUESTIONS

• Plural forms & Tentative Language!

How will you solve this problem? **vs** What are some ways you might solve this problem?
TIPS FOR POSING QUESTIONS

• Open-ended

Have you thought about...

vs

What is your thinking about...
TIPS FOR POSING QUESTIONS

• Positive presuppositions

Which of the strategies you know seem like a good fit for this problem?

As you want to confirm your answer is correct, what might you do?
TIPS FOR POSING QUESTIONS

• Higher-order thinking

What else can you tell me about that idea?  vs  How does that idea compare with others you generated?
TIPS FOR POSING QUESTIONS

• Higher-order thinking

What method did you like for this problem? vs When might you use that method?
Tips for posing questions

• Plurals
• Tentative
• Positive presuppositions
• Higher-order thinking
• Open-ended

POISING QUESTIONS FOR MATHEMATICAL PRACTICES
**Red Light, Green Light**

**WHO IS WINNING THE RACE?**

<table>
<thead>
<tr>
<th>Mary</th>
<th>Harry</th>
<th>Larrie</th>
</tr>
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<tr>
<td>$\frac{3}{4}$</td>
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<td>$\frac{5}{6}$</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Han</th>
<th>Miguel</th>
<th>Angela</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{5}{8}$</td>
<td>$\frac{5}{9}$</td>
<td>$\frac{2}{3}$</td>
</tr>
</tbody>
</table>

Fraction tells how much of the distance each player has already run.
What questions might you ask students about this task?

- Plurals
- Tentative
- Positive presuppositions
- Higher-order thinking
- Open-ended

**Red Light, Green Light**

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TCSM V3, p. 125
## Standards for Mathematical Practice

<table>
<thead>
<tr>
<th>Standards</th>
<th>Clarifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP.1. Make sense of problems and persevere in solving them.</td>
<td>MP.5. Use appropriate tools strategically.</td>
</tr>
<tr>
<td>MP.3. Construct viable arguments and critique the reasoning of others.</td>
<td>MP.7. Look for and make use of structure.</td>
</tr>
</tbody>
</table>

## Cluster: Extend understanding of fraction equivalence and ordering.

### Standards

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>KY.4.NF.1 Understand and generate equivalent fractions.</td>
<td>Students draw fractions and see equivalent fractions.</td>
</tr>
<tr>
<td>a. Use visual fraction models to recognize and generate equivalent fractions that have different numerators/denominators even though they are the same size.</td>
<td></td>
</tr>
<tr>
<td>b. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$.</td>
<td></td>
</tr>
<tr>
<td>MP.4, MP.7, MP.8</td>
<td></td>
</tr>
<tr>
<td>KY.4.NF.2 Compare two fractions with different numerators and different denominators using the symbols $&lt;$, $=$, or $&gt;$. Recognize comparisons are valid only when the two fractions refer to the same whole. Justify the conclusions.</td>
<td>Students use a variety of representations to compare fractions including concrete models, benchmarks, common denominators and common numerators. Note: Students determine which strategy makes the most sense to them, realizing they use different strategies for different situations.</td>
</tr>
<tr>
<td>MP.2, MP.3</td>
<td></td>
</tr>
</tbody>
</table>

Coherence: KY.3.NF.3d $\rightarrow$ KY.4.NF.2 $\rightarrow$ KY.5.NF.2
LESSON 11

Dividing Rational Numbers

11.1: Tell Me Your Sign

Consider the equation: $-27x = -35$

Without computing:

1. Is the solution to this equation positive or negative?
2. Are either of these two numbers solutions to the equation?

\[
\frac{35}{27} \quad \text{and} \quad \frac{35}{27}
\]
Credit Card Investigation: What is interest? (Day 1 of 4)

12th Grade Math » Unit: Exponential Functions and Equations

**Big Idea:** On day 1 students find percent increase/decrease and simple interest to establish a pattern which extends to writing exponential functions.

**Standards**

- HSN-Q.A.1
- HSN-Q.A.2
- HSN-Q.A.3
- HSA-CED.A.1
- MP1
- MP2
- MP3
- MP6
- MP8
LEADING FOR MATHEMATICAL PROFICIENCY
1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations**.
4. Facilitate meaningful mathematical **discourse**.
5. **Pose purposeful questions**.
6. Build procedural **fluency from conceptual understanding**.
7. Support productive **struggle** in learning mathematics.
8. Elicit and use **evidence** of student thinking.
Pose Purposeful Questions

SHIFT #5: FROM QUESTIONS THAT SEEK EXPECTED ANSWERS TOWARD QUESTIONS THAT ILLUMINATE AND DEEPEN STUDENT UNDERSTANDING
Make a trio or small group
QUESTIONING AND THE MATH PRACTICES

• Start at the top of the framework, then the Thinking Practices, and Finally the Supporting Practices.…

• Write 2 questions for each.
• Use the Questioning Tips!
• Be Ready to Share!
One possible list of questions for each Math Practice

**Answer Key for Tool 12.10 (Note: This is one possible set of answers.)**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>How is this task similar to a previous task you have completed?</td>
</tr>
<tr>
<td></td>
<td>What strategies might help you to solve this problem?</td>
</tr>
<tr>
<td></td>
<td>What helped you be successful in solving the problem?</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
<td>What expression or equation represents this data/situation?</td>
</tr>
<tr>
<td></td>
<td>Are these expressions equivalent? How do you know?</td>
</tr>
<tr>
<td></td>
<td>What do the variables/numbers/answer mean related to the context?</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Why did you use [a graph] to solve it?</td>
</tr>
<tr>
<td></td>
<td>How did you get [that equation]?</td>
</tr>
<tr>
<td></td>
<td>What do the rest of you think about Anna's strategy?</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
<td>How does your model (equation) connect to the situation?</td>
</tr>
<tr>
<td></td>
<td>Where can you find (the rate) in this situation? The table? The equation?</td>
</tr>
<tr>
<td></td>
<td>Are these two equations equivalent? Which (if any) is more efficient?</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
<td>How might a number line help you think about the problem?</td>
</tr>
<tr>
<td></td>
<td>What manipulative or picture might you use to solve the problem?</td>
</tr>
<tr>
<td></td>
<td>What other resources might help you with this problem?</td>
</tr>
<tr>
<td>6. Attend to precision.</td>
<td>When will this strategy work?</td>
</tr>
<tr>
<td></td>
<td>Which is the better unit of measure for this task?</td>
</tr>
<tr>
<td></td>
<td>What labels might be useful for this problem?</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
<td>When can you regroup numbers and maintain equivalence?</td>
</tr>
<tr>
<td></td>
<td>How might you use break-apart to solve this problem?</td>
</tr>
<tr>
<td></td>
<td>What is true about all of these triangles?</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td>What patterns do you notice across these problems?</td>
</tr>
<tr>
<td></td>
<td>How are these problems the same? Different?</td>
</tr>
<tr>
<td></td>
<td>How might this problem help you solve another problem?</td>
</tr>
</tbody>
</table>
What questions might you ask students about this task? ADD TO YOUR IDEAS

- Plurals
- Tentative
- Positive presuppositions
- Higher-order thinking
- Open-ended

**Red Light, Green Light**

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TCSM V3, p. 125
LET’S BE INTENTIONAL!

1. Identify the Math Practice

2. Identify questions

3. Remember questioning tips
<table>
<thead>
<tr>
<th>5 * 3 * 1</th>
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</thead>
<tbody>
<tr>
<td>• Individually, list 5 words that summarize this morning’s learning experience for you.</td>
</tr>
<tr>
<td>• Share your list with your table group and choose 3 words.</td>
</tr>
<tr>
<td>• Now select 1 that captures the essence of the morning for the group.</td>
</tr>
</tbody>
</table>
intentional

Adjective: Done on purpose; deliberate.
How tall is a stack of 25 of the ice cream cones pictured here?

2.25 cm

5.1 cm

- In a table record the heights of 1, 2, 3, ... cones.
- Write an equation to represent the pattern.

SMP#8: Now What?

✓ Use tables
✓ Look for patterns in the table
How tall is a stack of 25 of the ice cream cones pictured here?

2.25 cm

5.1 cm

How might you enhance the task to focus on a selected Mathematical Practice?
How tall is a stack of 25 of the ice cream cones pictured here?

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
How tall is a stack of 25 of the ice cream cones pictured here?

2.25 cm

Will 50 stacked cones be twice as tall? Why or why not?

SMP#2: Now What?

- Add a debate-type prompt.
How tall is a stack of 25 of the ice cream cones pictured here?

2.25 cm

5.1 cm

How tall would the stack of 25 be if the cone was the same height, but the rim was 1 cm?

SMP#7: Now What?

 ✓ Vary the values to explore the structure of the problem.

 ✓ In discussion ask, “What is changing? What is constant?”
COUNTING COUNTERS

Your whole set of counters is 12 counters.

1. How many counters are in one-fourth of a set?
2. How many counters make three-fourths of the set?
3. How many counters are in three-halves of a set?
4. How many counters are in five-thirds of a set?
5. How many counters are in ten-fourths of a set?
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• Which Math Practice might you pick?
• How might you enhance the task to press on selected Math Practice?
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What reasoning strategies are you using to determine the number of counters?

How are these problems the same? Different?

SMP#2: Now What?

• Ask questions about quantitative reasoning

TCSM V3, p. 112-113
COUNTING COUNTERS

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SMP#7: Now What?

• Ask questions about structure

TCSM V3, p. 112-113
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5. How many counters are in ten-fourths of a set?

How do the numerator and denominator relate to what you are doing with the counters (and why)?

How are you solving problems with unit fractions? Non-unit fractions?

SMP#7: Now What?

Ask questions about structure

TCSM V3, p. 112-113
COUNTING COUNTERS

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• SMP#2: Now What?
• Ask questions about quantitative reasoning

TCSM V3, p. 112-113
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SMP#8: Now What?

Ask questions about patterns

TCSM V3, p. 112-113
COUNTING COUNTERS

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What processes were you using repeatedly as you solved these problems?

What patterns do you notice across these problems?

How are these problems the same? Different?

SMP#8: Now What?

Ask questions about patterns

TCSM V3, p. 112-113
intentional

Adjective: Done on purpose; deliberate.
Your Turn!

WORK IN GRADE LEVEL GROUPS
1. Explore the task.
2. Discuss the mathematics with a partner.
3. Decide which Math Practice you will emphasize.
4. Discuss 1-2 adaptations for the task and/or questions to emphasize the Math Practice.
Clarise bets her friend Randall that she can read his mind and figure out his number. Randall silently picks the number 10. Clarise says:

- Choose a number
- Double it
- Add 9
- Add your original number
- Divide by 3
- Add 4
- Subtract your original number

Clarise says, “I can see that you got 7.” Is she correct? Prove that this is not mind-reading, but mathematics in action.
## Protocol for Enhance-a-Task

1. Explore the task.
2. Discuss the mathematics with a partner.
3. Decide which Math Practice you will emphasize.
4. Discuss 1-2 adaptations for the task and/or questions to emphasize the Math Practice.
OPENING TASKS: OPENING UP OPPORTUNITIES FOR MPS
8 EFFECTIVE MATHEMATICS TEACHING PRACTICES
FROM PRINCIPLES TO ACTIONS (NCTM, 2014)

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.
Opening Closed Tasks

Essential Question: How can we adapt a closed task so that it is an open-ended task?

TCSM V3, p. 60-62
OPENING CLOSED TASKS

BEFORE:  \[6 + 4 = ?\]

AFTER:

There are 10 fishing boats. Some are out fishing and some are on the beach. How many boats are out fishing and how many are on the beach?

[Is there another possible answer? ]
OPENING TASKS EXPERT GROUPS

• Review the task
• Brainstorm simple ideas for opening the task
• Select your favorite idea.
• Determine what Math Practice is supported with the opened task.

Front of Room
QUESTIONING + OPENING TASKS

With Intentional Focus on ONE Mathematical Practice
ENHANCING & IMPLEMENTING YOUR TASK

Which Reasoning MP (2, 7, 8)?

Which Supporting MPs 3, 4, 5, 6?

What specific questions might you pose?

Task:
MPs:
Task Adaptation (if any):

Questions:
OPENING TASKS EXPERT GROUPS

- Review the task
- Brainstorm simple ideas for opening the task
- Select your favorite idea.
- Determine what Math Practice is supported with the opened task.
1. Counting Counters (part 2)
2. Line ‘em Up
3. Find the Error
4. Greater, Less, Equal or Don’t Know
5. Property Lists
6. Predict How Many (2 tasks)
7. You be the Judge
8. Tilt or Balance (2 pages)
9. Can you Make it True

Tasks: CHOOSE ONE

Front of Room
GALLERY WALK
As you prepare quality tasks and plan good questions, what ideas from today will help you support students in developing the mathematical practices?
QR Code for folder with Activities from this session

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