A Pizza Delivery Problem
Cornell Math Explorer’s Club

November 7, 2020

1 Introduction: Everybody Wants a Piece of the Pie

Over the past decade, self-driving vehicle technology has greatly improved, making it possible for these devices to be used in everyday life. With that said, many people have been exploring various ways to use self-driving cars to help with food delivery. In 2019, Domino’s Pizza began using self-driving delivery cars to deliver pizzas in certain locations across the United States, and various grocery store chains experimented with the idea as well. Although the self-driving delivery vehicles led to an increase in business in some cases, there were still issues with the cost of running the vehicles, traffic safety concerns, and inability to meet demand in a timely fashion (just to name a few)\[1\].

2 The Problem: Coming up with a Recipe for Success

Inspired by Domino’s self-driving delivery car experiment, the local pizza shop in your neighborhood wants to use this new technology to help with their deliveries too! Here’s the catch - since self-driving delivery cars are expensive and customer satisfaction is their top priority, the shop owner wants to make sure that they are using these new cars in the best way. To do come up with a successful plan, they need your help!

\[1\]Acknowledgements: The topic, content, and problem for the November 7, 2020 MEC Session is inspired by and adapted from some of Mallory Gaspard’s joint work with Alexander Vladimirsky. Thanks, Alex!
Your Mission:

The shop owner has provided you with a map of the shop’s delivery region, along with a few important delivery locations marked. Your task is to determine the best delivery routes that at least meet the following criteria.

1. Minimize fuel usage

2. Minimize travel time from the restaurant to the customer to deliver the pizzas while they’re hot!

The car can only travel on official streets (straight lines on the grid), and the side of a grid square constitutes one link. The criteria above and the delivery map is simply a starting point. Throughout
the next section, we will build onto this framework to come up with the ultimate recipe for self-driving-car success!

3 Strategic Path Planning: A Delicious Idea

Before we dive into planning the delivery routes, let’s think about the information we will need to take into consideration during the process.

1. Is there any information about the vehicles, delivery process, customers, etc. that the shop owner did not give you that you think is necessary to take into account while determining the best delivery routes? If so, what information are you missing, and how might you use this information in your method?

3.1 Problem 1 - Less Fuel and More Pizzas!

The pizza shop has a few regular customers whose orders remain the same week after week. Since these are regular routes, the shop owner would like to identify the delivery route that minimizes fuel loss so that the self-driving car has enough fuel to make more deliveries throughout the day. Every week, one of the regular customers orders ten pizzas. Our first task is to determine the delivery route from the shop to the customer that requires the least amount of fuel.

Some Relevant Additional Information:

- While the car is on a highway (green links), it loses fuel at a rate of 1 liter per 8 miles.
- While the car is driving through the neighborhoods (black links), it loses fuel at a rate of 2 liters per 8 miles.
- Each link in the neighborhood map is two miles long.

Questions:

1. Are there any additional assumptions that you need to make about the car or the route in order to determine the best path? Discuss with your group.

2. Using your assumptions, outline the fuel-minimizing path to the delivery location. Is this route also the shortest (quickest) route from the restaurant? Discuss with your group your strategy for determining the route.

3.2 Further Considerations - Traffic Jams, Construction, Multiple Cars, Oh My!

Now that we’ve determined the fuel minimizing route for the shop’s regular weekly delivery, the shop owner has provided us with some updated information from the city regarding planned construction projects in Figure 2. The shop owner also told us that they plan to purchase two additional self-driving delivery vehicles. With this in mind, the neighborhood map below has been updated to reflect upcoming road construction projects by removing the links corresponding to street closures. The closed streets are represented by red links on the updated map.

Questions:

\[2\text{This roughly translates to a vehicle getting 32 miles per gallon on the highway which is a common rate for many modern vehicles.}\]
Figure 3: Map of delivery region with road closures marked in red.
1. How does the fuel-minimizing route change once the streets close? Discuss with your group.

2. Suppose that now you want to determine the route that is both the fastest and requires the least amount of fuel. How would you balance two objectives at the same time? Could there be situations in which there are multiple delivery routes that take the same amount of time and use the same amount of fuel? Which one would you pick? Discuss your ideas with your group.

3. Think beyond the one-car scenario. What are some strategies for coordinating delivery efforts between the three cars? What are other factors you might want to consider in your planning? Discuss these ideas and brainstorm model enhancements with your group.

4. Further Resources:

The math we used in today’s activity comes from path planning and optimal control. Path planning and optimal control incorporate ideas from many areas such as optimization, graph theory, calculus, and computing to determine the "best" or "preferred" way to do something. Path planning and optimal control problems arise in everything from determining flight patterns for commercial air travel, to determining the most effective way to administer medicine to cure diseases.

To start learning more about path planning and optimal control theory, the instructors personally recommend the following free hyperlinked resources:

Path Planning & Related Tools:

- Brilliant.org overview on shortest path algorithms
- Brilliant.org article about Dijkstra’s Algorithm to find the shortest path on a graph

Optimal Control Theory:

- An entertaining article about the history of Bellman’s Optimality Principle
- A detailed blog post on the Dynamic Programming Principle with code to follow along with
- Calculus of Variations and Optimal Control Theory by Daniel Liberzon (For those with some calculus experience)
- An Introduction to Optimal Control Theory by L.C. Evans (For those with some calculus experience)

References